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Conceptions on the teaching of subtraction: A study focused on an in-service teacher training course

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Abstract

This paper reports a research about a group of in-service teachers working in primary public schools in a poor urban zone in Monterrey city, Mexico. Its main aim was to study teachers' conceptions about the teaching of subtraction and, in particular, to know more about the role that they assign to context and contextualizing in the teaching process. Broadly speaking, the research arose from the interest to know more about the relationship between the training and education in mathematics of primary teachers and how the teaching and learning of mathematics actually takes place at school.

Key words: Teaching-learning, mathematics, elementary education, teachers' training.

Introduction

After more than a decade of the mathematics programs reform for elementary education in Mexico, there are few research studies which report on the changes or transformation in teachers' conceptions on the teaching and the learning of mathematics.

Even though the reform proposed a significant change in the focus on the teaching of mathematics in elementary education, which even led to the design of new free text books, we know, as shown by Llinares (1992), that the changes in the teaching of mathematics are not originated only by a normative curriculum, because teachers filter this curriculum through their mental framework, which includes mathematical knowledge, concepts and beliefs concerning mathematics as a discipline and its teaching process, as well as other aspects related to its role within the classroom. In this same sense, according to Ernest (2000), the empirical studies have confirmed that teachers' ideas, beliefs and preferences concerning mathematics influence the way they teach.

On the other hand, context plays an important role in the construction of mathematical concepts and procedures by the apprentice. Their importance lies, according to different researchers, such as Brousseau (1994), Charnay (1994), Carraher, Carraher & Schliemann (1995), Nunes & Bryant (1996), in giving those concepts and procedures meaning and, sense. For Brousseau (1994), for example, the teacher should first carry out the work in the reverse sense to the order followed by scientists, a re-contextualization and re-personification of knowledge: to search for situations which give meaning to the knowledge that is to be taught. According to those authors, many of the difficulties in teaching mathematics are caused because of the use of irrelevant contexts with little significance for the student.

We have chosen a basic subject from the mathematics curriculum for elementary education, the subject of subtraction, as a means to begin to the identify teachers' concepts on the teaching of mathematics. Teaching subtraction seems to us to be a mathematics subject of great interest for the purpose of our investigation, because it constitutes a subject where we can observe the school's resistance to the changes brought about by the different reforms in the mathematics curriculum for elementary education. Besides, it is one of the subjects in basic mathematics teaching which has been discussed the most in the permanent teacher training programs and which, however, continues to be a source for didactic conflicts.

To get a general overview of] the studies made about conceptions, beliefs and professional knowledge we reviewed the work of Thompson *Teacher's beliefs and conceptions: a synthesis of research* (1992) and the work of Fennema & Loef *Teacher's knowledge and its impact* (1992). In the same way, we have gone through the works of authors like Llinares (1996) and Llinares & Sánchez (1989); Carrillo (1998), Contreras (1999), Flores (1998), who have carried out studies about teachers' beliefs and conceptions regarding mathematics and its teaching.

The first stage of this research, reported on Martínez Silva (2001), is the most recent antecedent of this in paper. In that stage we were able to progress with the construction of a conceptual framework about the subject, as well as in the design and the validation of certain tools for information gathering that we have recovered for this second stage of the study.

On the whole research the main objective has been to study the elementary school teachers' conceptions regarding the teaching and learning of subtraction, particularly concerning the role of contextualization within this process.

The questions we wanted to answer through this study are the following: What conceptions do teachers have concerning the teaching of subtraction? What conceptions do they have about the role of contextualization in the teaching and learning process of subtraction? Which content or aspects of subtraction do they teach? Which aspects do they emphasize more? Why? What kind of situations/contexts do teachers use to give sense/meaning to the teaching of subtraction? In which way do teachers' conceptions relate to the aspects of subtraction they emphasize and to the didactic situations they propose?

Our position on the idea of conceptions takes the definitions used by authors like Ruiz (1994), cited in Flores (1998), Ponte (1994), Thompson (1992), Carrillo (1998), Contreras (2000), Fischbaun & Ajzen (1984), Moreno (2000), Furinghetti (1999).

For the purpose of this research we will understand the word *conceptions* as the whole of the internal representations evoked by a concept. They are the implicit organizers of the concepts, essentially cognitive in nature. They describe the nature of mathematical objects and their various images in the mind, regardless whether these are symbolic, graphic, etc.

The conceptions do not only refer to the nature of mathematics and mathematical objects, but also to the teaching and learning of mathematics. In order to achieve our research goals we will understand *conceptions on the teaching of subtractions* as the position taken by teachers regarding the purpose, objectives and teaching contents of subtraction in elementary education, the teachers' and students' role, the most appropriate type of teaching activity or instructional process, the role given to contextualization within the teaching and learning of subtraction, etc.

We will understand as *contextualization of teaching mathematics*, the process through which the teacher tries to establish relationships between the knowledge they have to teach and the situations in society where this knowledge will be used. In other words, the process through which a series of "real" situations of children's everyday lives, are transformed in mathematics problems in order to illustrate the mathematical concepts or procedures that have to be taught.

Research method

In the first stage of the study (Martínez Silva, 2001) over 100 elementary school teachers participated. In the second stage of the study a group of seven teachers and two elementary school teachers, with the following general characteristics:

- They work in public schools in the suburbs of Monterrey, Nuevo Leon, Mexico, with a low socio-economic standard of living.
- They teach 2nd or 3rd grade pupils in elementary school.
- The majority works a double shift.
- The majority has not participated for the last three years in a training course for the teaching of mathematics.
- The group's average age is 45.
- The average teaching experience is 24 years.

Based upon the methodologies of *case studies* (Llinares, 1994a, 1994b, 1998), and the *study of critical incidents* (Rosales, 2000), on the second stage of the research (Martínez Silva, 2003) a course for professional training (CPT) was design and developed. This course was called "The teaching of subtraction in elementary school", and it was useful for the gathering of preliminary information about the participating teachers. Also, three questionnaires were used as additional tools: open questionnaire, pondering questionnaire, ordering questionnaire. These were evaluated during the first stage of the study (Martínez Silva, 2001).

The CPT started from a case presentation, which is analyzed at the level of a small group with the conceptual support from both texts and the course teacher. The conclusions of each subgroup are registered in written form to be analyzed and discussed later by the group as a whole.

The Abel case (see Annex 1) is about a hypothetical pedagogical situation in which a teacher describes in writing, the difficulties in learning three of his pupils had in relation to:

- The solution of subtraction problems.
- The use of informal and non-conventional procedures to solve the subtraction problems.
- The difficulties met in the learning of the conventional algorithm for subtraction.

The questionnaires were analyzed in a qualitative and quantitative manner in order to compare them with the results of the first stage of the study (Martínez Silva, 2001). The information obtained during the CPT was gathered in three ways: audio recording of the working groups' discussion, audiovisual recording of the group discussion and written registration of the participants. Subsequently, these data were analyzed qualitatively and triangulated with the result of the analysis made of the answers to the applied questionnaires.

Results: some of the teachers' conceptions about the teaching of subtraction

In this section we show as an example, some of the conceptions of the teachers' group which participated in the study, about the subject on learning and teaching subtraction in elementary schools.

a) For the teachers, *contextualization of the teaching* of subtraction should be done through the posing and solving of problems presented in written sentences.

In the analysis made of the open questionnaire answers, we found that 26 of the 29 situations suggested by the teachers to approach the subject of subtraction can be found in the category of *problems posed in written sentences* which is equivalent to 89% of the presented problems. The three remaining situations were *written numerical calculation exercises* (11%).

It is interesting to observe that all the situations suggested by the teachers refer to written problems and numerical exercises, and how the approach to problems and exercises through other means of representation, as for example orally, graphic, through drawings or in a concrete manner, are absent. The previous results are similar to the ones found in Martínez Silva (2001), where the majority of the suggested problems were presented in a written form (85.5%), while the problems presented orally, through drawings, tables, graphs or concrete material were very few.

In the CPT course, the *problem set forth in writing* appears as the prototype of the "reasoned" problem:

> [...] Something happened yesterday: I gave the children a reasoned problem: "There is space for 500 pupils in the school. On Monday, 220 people enroll in school and another one 170 on Tuesday. You are asked: "Are there still places left in the school? How many are missing?". There we have two operations. They don't subtract; only one got the right answer [J-DE2.77].[1]

In the same way, the teachers point out that in order to help the learning of subtraction, different methods of representation, mainly concrete and through drawings, should be stimulated as a means to solving it. In their opinion, the concrete representation should be used only in the first two grades of elementary school, but, as of the second cycle, they should do without this aid for the comprehension and solution of problems.

> [...] In the first and second year, it is done with objects they can manipulate... [Ma-DE2.20].

> But in third grade, you have to take that away from them, because the children start to draw all over their notebooks and that can't happen in third grade… you have to stop this bad habit [C-DE2.21].

> And use concrete material… at the beginning [C-DE2.33].

> The concrete material is essential for the smallest children, so they can acquire knowledge [N-DE2.61].

b) It is essential for the teachers to place *keywords* in the text that describes the problem, because these can be used by the children as an indicator for the type of arithmetical operation they have to use in order to solve the problem.

According to the pondering questionnaire, the majority of the teachers agree that the problems we give to the children presented in written form should always have "keywords" which function as guides so they know what kind of operation they must use; however, there is a significant percentage that opposes this idea.

The previous results are confirmed during the case and critical incidents' discussion and analysis and where the teachers are in favor of offering "clues" in the posing of the problem in writing, which help the children to relate the problem to the operation needed to solve it. As can be read on the following:

> –[...] You sort of give them a clue, like "How much money does he have now?" to indicate it's a subtraction [C-DE6.13].

> –The keyword should be in the question, so they know that it's a subtraction or an addition, right? [Ma-DE6.14].

> –Change the question, right? to "How much money does he have left?" Right? [Ma-DE6.15].

> –Or "How much money has he got left?" [C-DE6.16].

> –No, he has left over [Ri-DE6.17].

> –So he knows they are going to give him back... that there is change left. Right? [Ca-DE6.18]

> –Or "How much change does he get back?" But that would even be longer. That expression of he has left over is… [C-DE6.19].

In the following excerpt from a discussion among the teachers, the role they assign to the keywords in the solving of a problem is shown with greater clarity.

> –[...] Look. You use words like *missing*, *take away*, *remain*. Those are words that a kid can understand [Ri-DE6.35].
>
> –Yes. They understand that something is left over. I have realized that, at least in my group, they understand better if I say, "How much have I left over?" They already know that that means after they have taken away the money, right? [Ma-DE6.36]
>
> –Well, for us maybe not, but maybe for the children it's easier to understand "How much do you have left over? [Ro-DG6.5].
>
> –I say yes [it's appropriate]. That it is handled like this in the groups. Yes, both concepts are used with the children. For example, at home, we simply say to them "How much do you have left?" "How much money do you have left?" [N-DG6.6].
>
> –Besides those two questions, what else could we ask? Or rather, what could we ask? [M].
>
> –How much money did you get back? [Ma-DG6.7].
>
> –Any other way of saying that? [M].
>
> –How much change did you get back? [N-DG6.8].

For the teachers there are words like *left over* which give stronger clues than the word *remaining* for the operation they have to solve. Questions like "How much money is left? How much money do you still have? How much cash did you get back? How much change did you get?" give bigger clues to the child that he or she is dealing with a subtraction.

In the same way, in the majority of the problems set forth by teachers in the Open Questionnaire, they used words that lead the children to think of the operation they have to apply. The most frequent words in the text describing the problem that leads the pupils to use subtraction are words like *missing* ("How much is missing?") and *lost*. Other words like *got out*, *gave away*, *escaped*, *sell*, *left over*, *less* and *difference* also provide indicators that link the problem to the operation which solves it.

Some examples for the use of keywords in the setting forth of problems in writing suggested by teachers in the open questionnaire are:

> [...] Luis has 185 marbles and he *lost* 62 in the game. How many marbles has he *left*?

> [...] Luis has 36 marbles and Antonio has 36 marbles, but they *lost* 24 marbles between the two. How many marbles do they have *left* all together?
>
> [...] Luis has 25 marbles. If he *lost* 13 during the game, how many has he *got* now?
>
> [...] In a vase there are 15 flowers; in another one there are 7. How many flowers are *missing* in the second vase in order to have the same number?
>
> [...] I want to buy a toy that costs $48. I have saved $25. How much do I *need* to be able complete the amount?

The previous results are similar to those found by Martínez Silva (2001) where words like *gave away*, *spent*, *ate*, *lost*, *stole* are also placed by the teacher as indicators in the written text to encourage pupils to use subtraction.

c) To teach the pupils how to solve a problem in the mathematics class means for the teachers basically, to set forth and solve problems that have the same type of related structure.

The prototype of subtraction problems that teachers propose to be able teach pupils the subject of subtraction in elementary school is found in the second category of the classification of additive problems (subtraction and addition) suggested by Vergnaud (1991), in which having given the children an initial measurement and its transformation, they are asked to find the final measurement. These problems can easily be represented through a direct action on a measurement and, therefore, they are easier to solve.

Some examples of problems which belong to the above category, proposed by the teachers, are the following:

> [...] Luis had 25 marbles. If he lost 13 during the game, how many marbles does he have now?
>
> [...] Luis had 25 stickers. If he gave 8 away, how many does he have left?
>
> [...] There are 25 marbles in a tin and 15 were taken out. How many marbles are left in the tin?

As in the case of the problems of the transformation of measurements, another large group of problems proposed by the teachers belong to the third category suggested by Vergnaud (1991) in which a relationship links two measurements:

> [...] Ruben wants to buy a ball costing $30 and he has saved $20. How much is he still missing?
>
> [...] Pepe collects matchboxes. He has 12 and he wants to have 25. How many is he missing?

> [...] Juan has 8 marbles and Pepe has 3. How many is Pepe missing to have the same number as Juan?
>
> [...] Hugo is 18 years old and Sergio is 6 years younger than Hugo. How old is Sergio?

The study of the place occupied by the unknown quantity in the problems suggested by the teachers provided interesting results. In the problems belonging to the second category, the unknown quantity is located basically in the search for the final measurement, given the initial measurement and the transformation. As for the third category of additive problems, in all of them, pupils have to find the relationship between the two measurements. Similar results were found in Martínez Silva (2001).

d) According to the teachers, the difficulties in learning mathematics are basically related to factors inherent to the pupils.

In the pondering questionnaire, most of the teachers consider that the children's problems in learning how to subtract are due to cognitive or concentration problems. In this sense, we observe how the teachers consider that children's learning difficulties in subtraction have to do with endogenous aspects of the pupil, such as cognitive or concentration problems, among others.

This position is maintained by the teachers in the ordering questionnaire in which the children's success or failure in the solution of subtraction problems is related primarily to the child's capacity to adequately relate the information of the problem. Secondly, to the child's confidence in his or her own procedures to subtract, as well as the knowledge of the vocabulary used in the text describing the problem. The previous results coincide again with those obtained by Martínez Silva (2001).

In the CPT, several factors, endogenous to the pupil, were also pointed out by the teachers as the being cause for the difficulties pupils have to learn mathematics.

Attention is a factor that appears to be closely related to the understanding, reasoning and solving of problems. The child's difficulty is found in his or her incapacity to detach him or herself from other stimuli and concentrate on the task, as if this were enough to comprehend and solve the problem, as can be seen in the following discussion excerpt:

> –[...] The problem could have been that the child hasn't read the problem attentively. He didn't concentrate on the problem [C-DE2.1].
>
> –This is a problem of attention or reasoning. The child knows the procedure; he/she arranges the units under the units and the tens under the tens, but the only trouble is that he added instead of subtracting them [Ma-DE2.5].

Very closely linked to the attention problem, there is the non-reading problem, which is an explanatory factor of the pupil's failure. According to the teachers, the child does not understand the problem because he or she does not read it attentively.

> –[...] The problem could have been that the child didn't read the problem carefully. He/she didn't concentrate on the problem [C-DE2.1].
>
> –The child didn't understand it. He/she didn't read it correctly [C-DE2.29].
>
> –But in this case, why do you think that the girl used an addition to solve the problem instead of using a subtraction? [M-DG2.7].
>
> –Because she didn't read the problem, because she didn't understand the problem. A lot of children are already solving the problem, before they have even read it. They get the numbers orally [Ma-DG2.8].

The lack of comprehension of the problem is another of the explanations set forth by the teachers regarding the pupil's difficulties to solve the problems.

> –[...] I say it's not a teaching problem; you are not going to teach the children that they have to subtract or add here. The child has to know through logic, what he/she is supposed to do [N-DE2.42].
>
> –I insist that it is not a teaching problem, but a reasoning problem. I am seeing that the child hasn't understood [N-DE2.46].

The lack of previous knowledge among the students, as well as the teacher's incapacity to reactivate in his or her pupils that kind of knowledge, is also mentioned as an explanation of the cause for the learning difficulties:

> –[...] We were saying that it was precisely that, that the teacher was not making use of the student's previous knowledge, that the teacher doesn't allow the students to create and solve subtraction problems and that the teacher doesn't use concrete material. And the recommendation is precisely that, that the teacher should use concrete material as a basis to teach mathematics. There lies the basis so the pupil can learn to solve problems [I-DG2.2].
>
> –I say it is a problem, because it is a problem of knowledge that the child doesn't have, or that he/she doesn't have and it shows up in the linguistic area when the pupil cannot interpret the text and therefore another procedure is applied here. What does this make you do? You have to go back, you have to interrupt your teaching plan and when you reach the higher levels, the problem is even bigger. We can go to the fifth or sixth grades and we can see that they are struggling a lot [J-DG2.11].

Conclusions

The teachers consider that the contextualization through the setting forth and solving of problems is fundamental in the learning and teaching of school mathematics. They assign some of the functions mentioned by Verschaffel & De Corte (1997) to the problems. In that way, they see in the setting forth and solving of the subtraction problems a means through which they can motivate the children to learn, to give the subtraction a meaning or to apply the calculating procedures they have learned.

In relation to the type of situations that should be set forth to the children during the process of teaching subtraction we have found certain inconsistencies. In some cases, the teachers consider that they have basically to provide the children with real problems or situations represented through concrete material. In a second approach, they consider representing problems and situations through drawings. Up to this point, this position coincides with Carraher *et al.* (1995) point of view; they point out the importance of providing pupils with everyday situations as a context for the teaching of mathematics. However, all the situations suggested by the teachers for teaching subtraction to children refer to problems set forth in written sentences and to numerical exercises. The setting forth of problems and exercises through other means of representation, either oral, graphical, through drawings or concrete material, is non existent.

The insufficient use of mathematical problems to be solved by using other means of representation has serious didactic consequences, because, as is pointed out by Verschaffel & De Corte (1997) and Vergnaud (1991), representation plays a fundamental role in the process of solving problems. This is why it is necessary on the one hand, to expose the children to mathematical situations or problems represented through oral, written or graphical narration or with drawings or concrete material. On the other hand, they should be encouraged to use different forms of representation in the process of solving problems. These two aspects complement each other and allow the children to learn to face very different types of situations and problems and to develop more flexible strategies for problem solving.

The information obtained concerning the type of problems suggested by the teachers to teach children how to subtract, as well as the use of “keywords” as indicators for the type of operation the children have to use to solve the problem, also have important educational repercussions, since the setting forth of problems plays a fundamental role in the teaching and learning of mathematics in elementary school. It is clear that there is little variety regarding the order and presentation of the information of the problems, as well as regarding the type of related calculation involved. The fact that pupils are exposed only to one or two of the categories of problems suggested by Vergnaud (1991), to the use of “keywords” and that the place of the unknown quantity is generally the search for the final measure in the case of transformation problems, limits the development of related calculation in the pupils and focuses them only on the learning of numerical calculations.

The teachers’ conceptions about the difficulties in the learning of subtraction coincides with two of the three great perceptions pointed out by Carraher *et al.* (1995) that have been used to explain the pupils’ failure to learn mathematics. For them, the pupils' difficulties to learn vary in nature: difficulties related to the pupil, to the didactic process, and hose linked to contextual aspects of practice. However, there is a tendency to explain the pupil's learning difficulties in solving the subtraction problems, based on causes pertaining to the pupil.

In a more general way, we can point out that we have found the coexistence between teachers with a traditional conception and those with a constructivist one, concerning the teaching of subtraction and mathematics in general. Besides that, there have been inconsistencies in the teachers' conceptions: for example, between the importance they give to contextualization and the type of situations they propose where didactic intervention is needed.

The inconsistencies found, as pointed out by Thompson (1992), seem to be the result of a complex relationship, whose influence springs from many sources:

- The teachers do not have the necessary skills and knowledge to apply the changes or reforms to the teaching of mathematics.
- The social context in which the teaching of mathematics takes place, imposes restrictions and offers opportunities.
- The teachers belong to a pedagogical group culture and tradition in which syntactic aspects are still favored over the semantic ones in the teaching of mathematics.
- The teachers adhere to teaching ideals that they cannot reached.

As a consequence, and coinciding with this author, we consider that the task to change the conceptions and mathematics teaching practices continues to be the main problem in the training of mathematics teachers.

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ANNEX 1. Abel’s case

Below, we present a situation concerning a permanent teacher training seminar. The seminar is related to the learning and teaching of subtraction. One of the teachers, Abel, who participated in the seminar talked about a situation he is currently going through with one of the groups of pupils he is working with.

Analyze the situation and give your opinion afterwards about the case described.

Abel, third grade teacher in an elementary school

Currently, I am working with a group of third grade pupils. This year, my pupils are having lots of trouble learning subtraction.

In previous years, I haven’t had these problems, because, generally, I used to start working with the same group of children beginning in second grade and continued to work with them in third grade.

I have a few cases like the ones that follow:

A) When I suggested the following problem, Ana solved it in the following way:

Luis has saved up $ 200 and buys a soccer ball that costs $ 125. How much money has he left?

$$\begin{array}{r} 200 \\ + \\ \underline{125} \\ 325 \end{array}$$

B) Compared with the previous problem, Carlos solves it orally following these steps:

> Luis has saved up $200 and buys a soccer ball that costs $125. How much money has he left?
>
> 200 - 125=
>
> “200-100 is 100”
> “100 –25 is 75”
> “Now he has got $75.”

C) Finally, there is Beatriz who solved the problem in the following way:

> Luis has saved up $200 and buys a football that costs $125. How much money has he left?
>
> $$\begin{array}{r} 200 \\ - \\ \underline{125} \\ 185 \end{array}$$
>
> “Five to ten 5”
> “Two to ten 8 “
> “two minus 1, one“
> “Now he has got $ 185”

Script of the case discussion

In relation to situation A.

a) Have you ever faced a similar situation?
b) Do you think that what is occurring in this situation could be due to a teaching problem? Why?
c) What do you think is the reason for Ana’s difficulty?
d) What recommendations would you give the teacher to help Ana in this situation?

In relation to situation B.

a) Do you consider that this situation is problematic? Why?
b) According to you: what is Carlos thinking about in order to solve the problem the way he did?
c) Do you think it is advisable for children to use informal or non-conventional strategies to solve subtraction problems? Why?

d) What would you do in such a situation?

In relation to situation C.

a) What is Beatriz's problem?
b) What would you advise the teacher to do to solve this situation?
c) Describe the subtraction procedure that seems to you to be the most appropriate to teach children in this school grade.
d) What do you think of the problem proposed by the teacher?

Abel's colleagues have suggested the following pedagogical intervention options:

Option 1

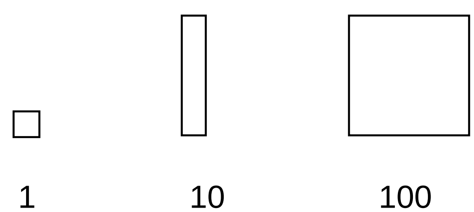
Use coins of different denomination as an aid so the children can solve the operation (200-125).

Option 2

Teach the children the conventional method for subtraction through a sequence of numerical exercises, gradually increasing the difficulty of the operation and the size of the numbers involved.

Option 3

Use multibase material as an aid for the comprehension of each step in the conventional subtraction procedure.



Option 4

Starting from the setting forth of problems related with the children's everyday life and then letting them free to solve it using either an informal procedure or the conventional one.

Option 5

Reinforce the learning of the conventional procedure for subtraction, explaining every step more clearly to the children and giving them many numerical exercises so they can learn to dominate it.

a) Among the didactic options described above, which seems to you to be the most adequate to teach subtraction? Why?
b) Which didactic option seems to you to be the least adequate? Why? What suggestion would you make to teach subtraction, but which, for some reason, is not , or cannot be, carried out in school?

[1] This code identifies each participation, including that of the teacher or moderator (M) during the course. First, the name of each participating teacher is identify by his or her name's initials. Second, the kind of participation is identify by two letters, DE for team discussion, DG for group discussions and CE for conclusions in writing. Finally, two digits represent the number of the activity within the participation takes place and the participation order on the discussion.