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Teaching Practice and Metacognition in High School to Promote Learning in Mathematics[1]

Práctica docente y metacognición en bachillerato para favorecer el aprendizaje de las matemáticas

Prática docente e metacognição no Ensino Médio para favorecer a aprendizagem da matemática

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Abstract

The objective of this article is to identify mathematics teachers' notions of metacognition and how it can be promoted in high school classrooms to support student learning. Two mathematics teachers working in Mexican upper secondary education (a high school) were interviewed following a guide designed around the obligatory educational model, conceptual knowledge, and teacher experience. The results show that participants have some basic knowledge of metacognition and promote planning, monitoring, and evaluation for a given problem, but fail to recognize these activities as metacognitive strategies or associate them with self-regulated learning. Further research is needed to gain deeper insight into the impact of teaching practice on the development and implementation of metacognitive strategies and self-regulation of learning in mathematics.

Keywords: mathematics teachers, metacognition, learning strategies, problem solving

Resumen

El objetivo de este artículo es identificar la noción que tiene el profesor de matemáticas sobre la metacognición y cómo promoverla en el aula de bachillerato para favorecer el aprendizaje en los estudiantes. Se entrevistó a dos profesores que imparten matemáticas en instituciones mexicanas de Educación Media Superior (bachillerato), para lo cual se diseñó una guía que tomó como referente el Modelo Educativo Obligatorio, conocimientos conceptuales y experiencia docente. Los resultados muestran que los participantes tienen conocimiento básico sobre metacognición y promueven la planeación, monitoreo y evaluación ante un problema dado, aunque no las reconocen como estrategias metacognitivas ni las relacionan con el aprendizaje autorregulado. Se requieren estudios que profundicen sobre el impacto de la práctica docente en el desarrollo e implementación de estrategias metacognitivas y

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la autorregulación del aprendizaje en matemáticas.

Palabras clave: profesor de matemáticas, metacognición, estrategias de aprendizaje, resolución de problemas

Resumo

O objetivo deste artigo é identificar a noção que o professor de matemática tem sobre a metacognição e como promovê-la na sala de aula do Ensino Médio para favorecer o aprendizado do alunos. Foram entrevistados dois professores que lecionam matemática em instituições mexicanas do Ensino Médio, para o qual foi desenhado um guia que tomou como referência o Modelo Educacional Obrigatório, conhecimentos conceituais e experiência docente. Os resultados mostram que os participantes têm conhecimentos básicos sobre metacognição e promovem o planejamento, o monitoramento e a avaliação diante de um determinado problema, embora não os reconheçam como estratégias metacognitivas nem os relacionam com a aprendizagem autorregulada. Requerem-se estudos que aprofundem sobre o impacto da prática docente no desenvolvimento e implementação de estratégias metacognitivas e de autorregulação da aprendizagem em matemática.

Palavras-chave: professor de matemática, metacognição, estratégias de aprendizagem, resolução de problemas

I. Introduction

Metacognition in mathematics has been studied for the last five decades (Desoete & de Craene, 2019) and focuses on how and why students regulate their knowledge in solving mathematical problems (Preiss et al., 2018). From a metacognition approach, solving mathematical problems enables the development of higher-level cognitive processes and solution strategies, while also enabling autonomy in learning (Schoenfeld, 2012; Rigo et al., 2010). A problem is defined as a "set of organized and oriented activities, with one or more solution strategies, in which it is possible to use a number of representations, allowing students to engage with mathematical activity" (Chávez & Martínez, 2018, p. 215).

On this basis, researchers like Apaydin and Hossary (2017) and Ricoy and Couto (2018) argue there is a need to study what teachers know about metacognition and how teaching practice generates self-regulation of learning. Dignath-van Ewijk and van der Werf (2012) state that mathematics' teachers conceptions of metacognition are reflected in their actions.

One way of strengthening metacognition in mathematics involves teachers striving for students to reflect upon what they do and what they learn, and become able to transfer their knowledge to other mathematical problems and their day-to-day life (Basso & Abrahão, 2018). In Mexico, this is one educational objective of the obligatory educational model (MEO, in Spanish) for high school mathematics teachers (Secretaría de Educación Pública [SEP], 2017). But what knowledge should or must teachers at this level of education have in order to achieve this, and what teaching tools are at their disposal to promote metacognitive strategies in their students?

A review of the literature shows that research has focused primarily on discussing the role played by basic-level education teachers in promoting metacognitive strategies in teaching and learning mathematics (Özsoy, 2011). Basso and Abrahão (2018) express a need to study teaching practice at later levels of education, where mathematics becomes more complex for students. In addition, as mentioned by Preiss et al. (2018), students at all levels of education need to self-regulate their learning if they are to control and build their knowledge. In this sense, this paper aims to provide a contribution to work on the impact of teaching practice on teaching and learning mathematics in upper secondary education, with the particular aim of reporting high school teachers' ideas of metacognition and its relevance in learning mathematics, and how, based on their experience, teachers foster metacognitive strategies in their students to achieve self-regulated learning in mathematics.

1.1 Metacognitive strategies in the mathematics classroom

The concept of metacognition refers to cognitive strategies and processes used to solve a mathematics problem, for example planning or selecting the solution process, monitoring, controlling the cognitive operation to solve it, and directing and evaluating the product (Desoete & de Craene, 2019). Another definition characterizes metacognition by students' *grasp* and *control* (in Spanish, *toma* and *control*, respectively) of a given problem (García et al., 2015), as explained below.

On the one hand, students are aware of their cognitive processes, the mathematical problem at hand and the solution process (Martínez, 2017). In this regard, Kilmenko and Alvares (2009, p. 18) assert that, at times, "when students tackle a problem, they understand that it relates to a topic they are unfamiliar with… that the way it is presented makes it difficult to understand… and that producing a chart will help them to visualize it and understand it better." For Schoenfeld (2012), this type of knowledge relates to the beliefs that individuals have about themselves, the problem, and the solution processes.

On the other hand, control is the active supervision of the organization and regularization of a student's knowledge using three metacognitive strategies: planning a solution process, monitoring, and evaluating that plan and its outcome. In solving a problem, students are expected to be able to develop an action plan, select the most viable procedure, monitor its implementation and control their actions, evaluate that implementation and determine if they followed the plan through, and also assess what they used and what they learned (Wulandari & Minarni, 2018).

Current interest in metacognition in mathematics has given rise to the development of pedagogies that focus their attention on learning as a process in which students are capable of self-regulating (Martínez, 2017; Wulandari & Minarni, 2018) and as a learning strategy that can be taught and modeled by teachers (Díaz et al., 2017). Indeed, Ellis et al. (2014) and Fourés (2011) claim that teaching practice can be used to promote and strengthen three metacognitive strategies in learning mathematics: planning, monitoring, and evaluation. Each of these strategies is described below:

- Planning is associated with determining the solution process for a mathematical problem and anticipating the activities to be undertaken (Fourés, 2011). One way to promote this in the classroom is through questions that lead students to think and establish the process(es) necessary to solve the problem, indicate whether the instruction is clear, and identify key information in the problem to determine the solution procedure (Ellis et al., 2014).

- Monitoring consists in supervising and rectifying the implementation of the process (Jaramillo & Simbaña, 2014). To bring about this strategy, teachers can stimulate student interest in comprehending the full process up to the point at which the solution is found, with the goal of identifying any errors in execution and making adjustments so as to change or validate the process and its implementation (Osses & Jaramillo, 2008).

- Evaluation means assessing the solution process, contrasting the results, and determining if they have been effective and correct (Fourés, 2011). To develop evaluation in the classroom, teachers should encourage students to verify and argue whether the process and solution to the problem are correct, and determine if the right plan was designed and if it was implemented in the right way, while also guiding students to make generalizations about what was learned and identify other similar problems that can be solved following the same plan (Özsoy & Ataman, 2009).

Authors such as Rigo et al. (2010) report that while there is a wide range of studies focused on describing metacognition in students, few explore the role of teachers and their adherence – as one factor – to metacognition and self-regulation of learning in mathematics. They also note the existence of fewer studies on teachers' conceptions of metacognition, particularly at high school level.

1.2 Teaching metacognition under official policy

Education policy in Mexico follows the Obligatory Educational Model (MEO, in Spanish) (SEP, 2017), the aim of which is to promote "knowledge, skills, attitudes, and values that enable lifelong learning in students" (p. 11). It also provides direction for the practices of all those involved in national education, including high school teachers. The MEO proposes that students should develop awareness of their learning process, in terms of controlling how and what they learn. The objective of the policy is therefore for students to develop and improve their metacognitive processes such that they become autonomous learners.

To achieve metacognition, the MEO requires mathematics teachers to create spaces where students reflect on their learning and any challenges they may encounter, with the aim of avoiding or correcting them. In other words, students are led to develop awareness, control their learning and build knowledge. In this regard, the MEO proposes that "schools accommodate self-regulation… to promote the development of knowledge" (SEP, 2017, p. 98).

This means that high school mathematics teachers need to know what reflection and self-regulation of learning is, and how it can be brought about in their students. The MEO considers that reflection occurs in terms of metacognition and refers to students' grasp and control of any difficulties. Reflection is seen as a process in which students "think about thinking in order to develop a strategy," leading them to question what is needed and identify the knowledge they already possess, so that they can gain insight into how they learn and for what purpose (Wulandari & Minarni, 2018, p. 32). Self-regulation, meanwhile, refers to students' capacity for control over "their thoughts, actions, emotions, and motivation through personal strategies to achieve the objectives set" (Panadero & Tapia, 2014, p. 450). In this sense, self-regulation leads students to become autonomous, active learners who control their actions – planning, monitoring, and staying focused, for example – to solve a given problem (Basso & Abrahão, 2018).

II. Method

This study can be described as qualitative and descriptive in scope (Stake, 1994). We use a case study methodology to illustrate, through representative examples, how teaching practices promote, deliberately or otherwise, metacognitive strategies in the mathematics classroom. This method seeks to describe and understand what is unique about the teachers participating in the study, but also what they have in common in terms of their practice in relation to metacognition.

The research was conducted in Mexico and involved two high school teachers who work at the same school and teach a geometry and trigonometry course to different groups. We decided to perform the study with these teachers because unlike others, they expressed, at different points of the research, great interest in bringing about autonomous learning in their students. They also fulfilled the selection criteria: they participated willfully in the research, they had a background associated with the course taught, they were mathematics teachers at a public high school preparing students for a general high school diploma (bachillerato general), they were classroom teachers, and they had knowledge of the MEO policy (SEP, 2017). In line with these criteria, the two teachers – identified as Bruno and Esteban – had received training in mathematics, had at least eight years' teaching experience at high school level, and were certified and evaluated in teaching competencies for upper secondary education.

As part of the participant selection criteria, we considered that high school teachers were responsible for "facilitating the educational process by designing meaningful activities that promote the development of competencies (knowledge, skills, and attitudes), [and must also] empower students in their role as autonomous managers of their own learning" (Subsecretaría de Educación Media Superior, 2017, pp. 5-6). In this sense, regardless of teachers' initial background or years of experience, or the number of students they have, teachers must fulfill their responsibility and commitment as educators.

Data was collected through interviews. To this end, a semi-structured interview guide (Abero et al., 2015) was prepared for the two teachers, based on the MEO and any conceptual knowledge and teaching experience in relation to metacognition and its promotion in the mathematics classroom. The interview guide was validated using triangulation (Wulandari & Minarni, 2018), and was first submitted for review by two experts in mathematics education and then piloted with two teachers external to the study, but who exhibited the same academic characteristics as Bruno and Esteban – notably, they had training in mathematics, taught mathematics at high school level, and were familiar with the MEO.

The interview focused on the meaning of metacognition and planning, monitoring, and evaluation strategies, as well as the scope of teaching practice aiming to foster these strategies in students (see Table 1 and Appendix). Bruno and Esteban were interviewed separately to prevent any influence of one over the other. The interviews were recorded and had an average duration of 60 minutes.

Table 1. Guiding ideas in the interview guide for participating teachers

Guiding idea	Objective
Knowledge of the Obligatory Educational Model	Explore what the teacher knows about the requirements of the policy in relation to metacognition, reflection, and self-regulated learning.
Conceptual knowledge	Explore what the teacher knows about metacognition and self-regulated learning.
Teaching experience	Explore the teacher's practice, based on his teaching experience, in terms of promoting metacognitive strategies.

The information collected in the interview was analyzed based on inductive categories (Miles et al., 2014). With that aim, the recordings were transcribed and the teachers' answers were grouped into three categories (see Table 2), which relate to 1) the meaning of metacognition and the requirements established by the MEO to develop metacognition in students, 2) teacher actions aimed at fostering metacognition in the classroom, and 3) types of strategies deployed in response to a mathematical problem.

Table 2. Inductive categories for the analysis of information collected in the interview

Category	Indicators
1. Metacognition based on the Obligatory Educational Model.	Requirements or demands made of teachers.
2. Metacognition according to the teacher.	Similarities/differences between reflection, self-regulation, and learning to learn. Teacher actions to promote metacognition.
3. Metacognitive strategies in mathematics.	Planning: Organizing and analyzing the problem. Monitoring: Choosing a strategy, operations implemented. Evaluation: Explanation and justification, value judgments.

Our analysis made it possible to identify similarities and differences in what metacognition means to teachers, its relationship with the concepts of *reflection*, *self-regulation*, and *learning to learn*, how they interpret the requirements of the MEO, and the scope of practice aimed at bringing about metacognition in their classes. The results are presented below and are based on the three categories and their respective indicators.

III. Results

3.1 Metacognition based on the MEO

For these teachers, metacognition is associated with reflection, self-regulation, and learning to learn. In this sense, we identify two aspects of metacognition: a) the application and meaning of mathematical content in student daily life; b) the importance of students recognizing their active engagement and developing awareness of their responsibility in order to learn. This is reflected in Bruno and Esteban's answers in relation to reflection on learning and learning to learn:

> … anything learned should be applicable to life… the MEO says that if we're teaching the Pythagorean theorem, students should know where it can be applied. (Bruno)

> Students need to learn and understand ways of learning. The ways they manage or find it easier to build knowledge. (Esteban)

From Bruno's perspective, when the policy refers to "in-class reflection," this means students should, given contextualized exercises, use their prior knowledge to determine if the mathematical content covered in class can be applied to other contexts. Bruno also stresses the need to design teaching techniques that stimulate such a reflection in students: "Designing and applying teaching techniques by way of which they can… converse with other classmates or their teacher, think, solve, reflect on what happened, and in so doing construct their learning."

Esteban, meanwhile, notes that different activities can be used for students to recognize – reflect on – how learning is made easier for them. Esteban believes that high school mathematics teachers' efforts should be geared toward making "students center stage and the most active part of knowledge." Likewise, he recognizes that the objective of the policy is to develop students' ability to understand ways of learning and what they need in order to learn:

> One of the competencies [based on the MEO] is student self-knowledge… and students should understand the ways in which learning becomes easier for them. Know and learn study techniques, see which they find most helpful, and… develop their skills so they can continue to further their knowledge.

3.2 Metacognition for high school mathematics teachers

Reflection, self-regulation, and learning to learn: similarities or differences. The results presented below illustrate how high school teachers identify certain characteristics of metacognition, based on their explanation of self-regulation, learning to learn, and reflection on one's own learning (Table 3).

Table 3. Metacognition as reflection, learning to learn, and self-regulation

Teacher	Meaning of metacognition	
	Self-regulation	Reflection and learning to learn
Esteban	Students knowing what it is they want to learn, what they'll use it for, and how they learn.	They refer to the same thing… if I think about the ways I'm learning, I'm learning to learn.
Bruno	Students arriving at knowledge and using their own methods and techniques [to learn]. It's being aware of what they're learning.	[Reflection]: Whether what was learned is well learned. [Learning to learn]: Knowing or being familiar with what they learned.

Both teachers believe that self-regulation, learning to learn, and reflection all bear a relation to one another and to metacognition. For Esteban, by reflecting, students learn to learn and thus understand what is helpful to them and what they need in order to learn – for example, what study techniques they find most effective, how much time they have, and what information they need to solve a given problem. Bruno, meanwhile, notes that "one leads to the other"; in other words, for self-regulation to take place, students need to reflect on and recognize the options to solve the problem, become aware of the mistakes they make, and determine which procedures are more viable than others to solve the problem.

This suggests there are similarities between the two teachers in the types of strategies associated with metacognition. For Bruno, planning can be recognized as the generalization of mathematical knowledge and the functionality of knowledge gained. As an illustration, Bruno gives the following example:

> In one session we went out to the yard to measure the height of a building. The students asked me how we'd do it with a triangle and a straw, based on their own shadow and height. I gave them the following instructions… The bottom line is that some students did follow the instructions and calculated the approximate height of the building, and realized that we used several topics we'd studied before, like Pythagoras.

Also, for Bruno, monitoring produces alternative ways to solve a mathematical problem:

> On the topic of trigonometric functions, we started recalling the Cartesian plane and I asked them questions – because they do know [the location of the quadrants] but they haven't processed it properly. It's like a jigsaw puzzle that the teacher has to help them put together. And they said, "I know one is X and the other one is Y." Then you ask them questions. You don't necessarily tell them, "There's the Cartesian plane, with four coordinates." No. You didn't allow the students to find out if what they knew was right or wrong. Rather, you ask them, "What do you think is the first quadrant?" And they might say, "I think the positive one, because that's where you start plotting the graph."

For this teacher, evaluation concerns the methods or procedures used by students to arrive at a correct answer:

> … the hope is always that the idea comes from the students themselves. For example, I'll ask them, "How would you solve it?" If they still haven't solved it after so long, I tell them, "Look, guys, there's this," and then they start putting it into practice. When there's a mixture of topics, they put forward an idea, and if the path to the solution seems too difficult or long to them, I say, "Well, there's the law of cosines and sines," and they consider it and decide how to proceed.

Notable in the previous excerpts is Bruno's idea of stimulating students' autonomy to use prior knowledge, and prompting them to take on mathematical tasks themselves.

As far as Esteban is concerned, reflection on learning is achieved when students identify what they know and do not know, when they recognize whether or not they need help and with what. In this respect, he comments:

> There are times I'm the one guiding the class, and other times when I say, "Do, do and ask." [In those types of activities] they realize what they're able to do on their own, or they ask questions about the steps, and when they've worked it out, they want to take part… [to explain], and the rest of the group notices other ways of solving it.

For this teacher, solving a problem is associated with carrying out the process directly:

> I tell the students: "Do it and we'll take a look tomorrow." The next day, you check and guide them to explore how they thought about it and did it. So a student who's active in class, even if that student's just letting others have a say, is in some way thinking about the approach that should have been taken.

Esteban believes that when students solve a problem, they may activate a number of metacognitive strategies, analyze the characteristics of the problem and determine its value in order to solve it, thus establishing how much attention and effort is required. If the teacher guides the student at this point, this will achieve a greater impact on the interest given to and value placed on the problem.

Metacognition from teaching practice. Presented below are the participants' ideas concerning how to achieve metacognition in the mathematics classroom. Bruno notes that when students are faced with a mistake in solving a problem, they should know the concepts – understand, theoretically, what they mean. He explains that that helps students to master the application of content covered in the classroom, and properly allocate time to solve the problem. To this end, he asks students to:

> Research concepts; when we encounter them in a problem, I tell them to get their glossary out and have a look – here, it says if this is like this, we do it like this… I tell them they must know the concept to understand and carry out the processes properly.

Reminding students of what they need to know and what they already know enables them to develop a plan, based on their definitions, to apply this in their working. Bruno also believes student metacognition is supported when he presents them with different options to understand the problem and learn. He says:

> Students retain knowledge if we as teachers are able to step in and ask the right questions. Some problems are such that they can arrive at the result with the knowledge they already have. But we tell them, "but there's another process whereby you can get the same result using fewer steps." And yes, many students do say, "But I understood and have already figured out the first one." The idea is that they decide which process they understand and which is best for them.

Esteban is of the view that cognitive self-regulation is associated with two types of evaluation: self-evaluation and co-evaluation. He believes that his classrooms create spaces for both types of evaluation to take place. Self-evaluation means that students reach a consensus among themselves about what is right or wrong in a mathematical problem, and as part of that process, they self-regulate what they know and become aware of any mistakes they have made. As for co-evaluation, he associates this with the student's ability to identify the different procedures to solve a problem.

3.3 Metacognitive strategies in mathematics

For Bruno and Esteban, planning, monitoring, and evaluation strategies enable students to reflect on and control their learning in mathematics. Presented below are the results in relation to these metacognitive strategies.

Determining a plan of action in response to a mathematical problem. Both teachers agree that planning plays a crucial role in determining how to solve a problem. For Esteban, planning involves taking ownership of the content covered in class in such a way that students customize the way they use what they learn in mathematics to solve problems, while Bruno points to a need for students to design action plans to guide them in determining what to do to solve a problem.

Planning allows students to become autonomous in terms of defining how to solve the problem. In other words, the responsibility for solution processes should be placed on students. But Bruno reduces this autonomy to a consequence of having a reward (such as a high grade), noting, "Teachers endeavor to do as little as possible so that it is the students themselves who do it, and they have no choice but to do it, [even though it may only be] to earn a reward, in this case their grade." Bruno outlines what high school teachers can do to encourage students to plan:

> Ask guiding or directed questions, in addition to presenting the context of mathematical problem; for example, in a right triangle, how can we solve – what can I do to find the length of one side, and they answer, "Oh! Well, there are only two [ways], there's Pythagoras and trigonometric ratios."

This shows that, as far as Bruno is concerned, asking these types of questions allows students to propose steps to solve a problem and "start to take ownership of whatever will help them, whatever they're going to use," while applying and remembering what they know. To illustrate this, he notes:

> [The Pythagorean theorem] uses right triangles that have two legs that will always measure less than the hypotenuse. So some students say to me, "I did it and got this length." And I'd reply, "How is a leg going to be longer than the hypotenuse?" That's when the students themselves answer, "Oh! The definition of a hypotenuse says that it's the longest side of a right triangle."

The approach described by Bruno is viewed by Basso and Abrahão (2018) as an interesting way of presenting and solving mathematical problems that takes on relevance for high school students, as it leads them to put into practice their previous knowledge. From this perspective, asking questions or proposing problem situations encourages the use of different cognitive resources, such as remembering and applying definitions and processes in problem solving.

Esteban, meanwhile, considers that planning is achieved when students integrate, reflect on and recognize the application of mathematics in other tasks. This happens when students are presented with a situation requiring the application of knowledge. From this perspective, through planning, “the aim is for students to build their skills and knowledge so that when they need to apply them, they are able to integrate everything, because they reflect on and process it differently so they can apply it; they customize it, so to speak.”

Monitoring: reviewing and defending solution processes. Both teachers associate the monitoring strategy with reviewing, defending and justifying the solution process for a mathematical problem. For Esteban, it is important for students to know what they are doing and to “reason properly, so they can discover if they’re managing to generate the necessary knowledge” to learn mathematics. Furthermore, one opportunity teachers have to ascertain whether students are monitoring their learning is through the answers they give to questions about hypothetical situations. Esteban notes that questioning students – for example, in the case of right triangles, asking them what would happen if the leg lengths were added together – leads them to put forward arguments and ultimately realize the claim they are making.

Esteban also frames monitoring as a “small self-evaluation” in which students are able to determine if the plan or process they have designed as a solution is correct, and put forward arguments to support the steps followed to solve a mathematical problem. In this regard, he states:

> The teacher’s job is to present situations that lead students to think and [decide] if they’re sure about how they’re processing the information, if they’re doing it correctly. You tell the students… you either confirm the answer, and they self-evaluate and say, “Oh, OK, [good]! My thinking was right,” or you tell them what would happen if they did that, and then automatically they come to the answer, without you giving it to them.

Both teachers agree that having students suggest how a problem can be solved is an example of monitoring, as after the teacher explains certain concepts or solution methods, they start to apply and use the methods they find least complex. This is in line with Fernández-Gago et al. (2018), in considering that teachers of mathematics should guide students to have more or better structured specific mathematical knowledge and use different metacognitive strategies to solve problems, and enable them to use specific knowledge and their own strategies to solve problems.

Evaluate solution processes. For Esteban, evaluation, like self-regulation, is associated with the types of formative evaluation. He states that each teaching unit should include one form of evaluation, and that “each semester should have a self-evaluation and a co-evaluation.”

Esteban believes that metacognitive evaluation, like assessing the application of the plan and the execution of the problem-solving process, occurs when students self-evaluate and co-evaluate. In this sense, Esteban makes reference to the use of checklists and rubrics that students answer. “What are the processes students should have followed to solve the problem? Did they complete this part or not? So students realize where they’re making a mistake.” In the co-evaluation, teachers ask students to check other’s work, and for each student to:

> ... indicate the mistakes; tell [his or her classmates] where they went wrong and what part of the process is incorrect, obviously under the direction of the teacher; check the process to see if they completed a series of steps; identify what the problem is asking and identify the information being offered by the problem, the process, and the solution.

In Esteban’s opinion, evaluation is encouraged when after reviewing mathematical content, teachers employ activities that involve evaluating the processes followed by students to solve problems associated with the content.

Bruno believes that evaluation is encouraged through an activity he calls “confrontation” (*confrontación*), which aims to have students discuss the solution processes for a problem so they can become aware of any mistakes they have made and explain them, with the goal of avoiding them in similar problems. The activity also helps students to propose and reflect on other types of solution processes. Bruno offers up the following remarks on the context and instructions for this activity:

> I don't explain anything to them, I give them time… After that time, I take in their exercise books and ask them who wants to come forward and solve it and explain how they did it. So then they see what their mistakes were, why they made them, and why they won't make them again…. That's when [teachers] use confrontation [because the students say to me], "Excuse me, sir, but I used this other method and I got the same answer, is that OK?". Yes, we don't tell them, "We didn't study that." The point is that [students] use any means to arrive at the result.

Bruno's input shows that teachers provide their students with opportunities to review their working and answers, and if necessary, make adjustments. Teachers ask questions and evaluate students' level of self-regulation and understanding of mathematical procedures, but as noted by Apaydin and Hossary (2017), it is crucial that they impress upon students the importance of knowing and putting into practice learning strategies that they find helpful, as this allows them to master the strategies and put them to use in similar problems and other mathematical contexts.

IV. Discussion and conclusions

The results show that the teachers who took part in the study have an idea of metacognition and how it can be generated in the mathematics classroom. Similarly, the meaning of planning, monitoring, and evaluation is clear to them, but this meaning differs from what has been reported in the literature on metacognitive strategies.

Bruno and Esteban provide an account of the role they play as teachers. For them, their teaching practice is important and contributes to their students' learning process, and they recognize themselves as facilitators who are committed to improving their practice in teaching high school mathematics. But the results show that neither teacher identifies tools or suggestions to achieve self-regulation in the classroom. The results obtained suggest that, in addition to outlining the requirements demanded of teachers, the MEO, as a guide to teaching practice in the classroom, should also provide information on teaching theory and practice to lead students to take an interest in and employ metacognitive strategies as they learn mathematics.

According to Schoenfeld (1985), metacognition can be brought about in students by asking them questions about planning, monitoring, and evaluating in mathematical problem solving: a) "What exactly are you doing?" so that students describe the process more precisely; b) "Why are you doing it that way?" so that students describe how what they are doing relates to the problem's solution; and c) "How does it help you?" so that students explain what they will do with the result when they have it. Although in the cases reported in this study, the teachers do not make direct reference to these questions proposed by Schoenfeld, the insights they provide do give an indication of how they promote these strategies in their high school students.

The objective of this study, consistent with the method employed, is to show specific examples of the reality in classrooms from a perspective of what high school teachers know about metacognition in mathematics. The results suggest a need for research focused on what happens in undisturbed environments or vulnerable contexts (Páez et al., 2020), as is the case with a high school mathematics classroom. Such research should describe a) teacher actions that can be identified as generating metacognition, and b) which metacognitive strategies teachers foster in their students within the mathematics classroom, based on their teaching practice.

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Appendix. Interview guide

The purpose of this interview is to gain insight into your experience as a high school teacher.

SECTION I: Knowledge of policy

The Obligatory Education Model (MEO), which serves as a foundation for the high school curriculum where you work, includes the following statements:

- "A key component of lifelong education is reflecting on the ways in which one learns, and on some skills, like memory or attention, that can be used to fine-tune and improve learning."
- As part of the teaching principles, "school accommodates cognitive self-regulation… to promote the development of knowledge."

With respect to these statements:

1. What does the model mean when it says that students should reflect on the way they learn and on their difficulties, and what is the purpose of this?
2. What are the characteristics of this type of reflection, and under what conditions does the model expect this reflection to take place in mathematics?
3. What actions or strategies does the model expect of mathematics teachers in order for reflection and cognitive self-regulation to occur in students?
4. Based on this educational model, how can cognitive self-regulation serve as a basis to promote the development of mathematical knowledge in students?
5. What teaching tools or suggestions does this model provide to mathematics teachers in order to achieve this reflection and cognitive self-regulation in their students?

The school's curriculum states that "teaching is centered on student learning. This means that students should learn to learn." In this regard:

6. What does the curriculum mean when it says that teaching means students should learn to learn?
7. What is the purpose of students learning to learn, in mathematics for example?
8. What is required of mathematics teachers by the curriculum in order for such learning to occur in students?
9. What teaching tools or suggestions does the curriculum provide to mathematics teachers in order to achieve this learning?

SECTION II: Conceptual knowledge

1. The MEO makes reference to the concept of "cognitive self-regulation." What does this concept mean to you?
2. What are the characteristics of cognitive self-regulation in the context of teaching mathematics?
3. What student actions or strategies demonstrate that self-regulation is taking place?
4. What is the purpose of students learning to learn?

The MEO and the school's curriculum mention the following concepts: a) reflection on one's own learning, b) cognitive self-regulation, c) learning to learn, and d) control of learning. With respect to this:

5. Do you believe there are any differences or similarities between these concepts? Please justify your answer.

6. What conditions are required in order for students to learn to learn, and to achieve cognitive self-regulation, reflection on learning, and control of learning?

SECTION III: Procedural knowledge

1. What do you do when students make conceptual or procedural mistakes during a mathematics class?

2. In your class, are students given an opportunity to defend, justify or explain the procedures or answers they arrive at for a given problem? Why? How does this happen?

3. When you set a mathematical problem, are students told how to solve it or do they have to come up with their own proposal? What is the purpose of this?

4. How can students be made aware that what they have learned in class is applicable to other similar mathematical problems?

5. In your classes, are there times when students are asked to review (evaluate), either individually, in groups or with the whole class, the contributions they make or their procedures or answers to a given problem? What is the purpose of this?