Revista Electrónica de Investigación Educativa

Vol. 10, No. 1, 2008

Previous Knowledge, Mental Models and Problem Solving. A Study with High School Students

Conocimiento previo, modelos mentales y resolución de problemas. Un estudio con alumnos de bachillerato

Joan Josep Solaz-Portolés
jjsolpor@telefonica.net
Centro Tomás y Valiente
Universidad Nacional de Educación a Distancia

IES Benaguasil (València)
46180, València, España

Vicent Sanjosé López
vicente.sanjose@uv.es
ERI-Polibienestar
Departamento Didáctica Ciencias Experimentales
Universitat de València

C/ Alcalde Reig, 8
46006, València, España

(Received: July 10, 2007; accepted for publishing: February 25, 2008)

Abstract

In this paper an experiment was carried out to test the theory of mental models of Johnson-Laird, which classifies subjects according to their previous knowledge. The subjects of the experiment were high school students of Valencia (Spain), to whom a problem solving test was administered. The results found seem to confirm that an inverse relation between the number of mental models implied in the problem and the percentage of subjects that solve it correctly, as the theory predicts. Moreover, subjects of higher previous knowledge do not always solve problems significantly better.

Key words: Cognitive processes, structure of knowledge, problem solving.

Resumen

En este trabajo se llevó a cabo un experimento, para poner a prueba la teoría de modelos mentales de Johnson-Laird, según la cual los sujetos se clasifican según su conocimiento previo. Los sujetos participantes en el experimento fueron alumnos de bachillerato de un centro educativo de Valencia (España), a quienes se les administró una prueba de resolución de problemas. Los resultados parecen confirmar un relación inversa entre el número de modelos mentales implicados en el problema y el porcentaje que lo resuelve correctamente, tal y como predice la teoría. Además, los sujetos con mayor conocimiento previo, no siempre resuelven significativamente mejor los problemas.

Palabras clave: Procesos cognitivos, estructura del conocimiento, resolución de problemas.

Introduction

The works of Santamaria, Garcia-Madruga and Carretero (1996), and Garcia-Madruga, Gutierrez, Carriedo, Moreno and Johnson-Laird (2002), are two samples highlighting the importance of mental models in human reasoning. More specifically, these studies emphasize the potential of the theory of mental models proposed by Johnson-Laird (1983, 1990, 1996 and 2000), which is based on the assumption that the mind constructs internal models of the external world, and that it uses these models to reason and make decisions. Each mental model represents a possibility in reasoning and understanding of events, situations or processes, and the mind reproduces those by capturing their most characteristic elements and attributes. Mental models can represent relationships between three-dimensional or abstract entities; these can be static or dynamic, and can serve as a basis for images, although many components of the models may not be visible.

Unlike propositional representation, mental models have no syntactic structure: they are depictions that reproduce by analogy, the structure of what we are trying to represent. However, representations may be used in mental models in the form of propositions or images. Representations are not durable in the long-term memory, as are patterns of knowledge; mental models are constructs realized with the data the individual perceives at a specific time, i.e., that are processed in the short-term or working memory. It is noteworthy that, for this theory, the number of models is the main obstacle to syllogistic reasoning. In fact, those issues in which it is necessary to generate two or three mental models are more difficult to resolve than those in which only one is required (Johnson-Laird and Bara, 1984).

Prof. Marco A. Moreira has drawn on this theory in his school of thought regarding the teaching of science. This author, in an introductory article which focuses on mental models from the perspective of Johnson-Laird's theory, aims to provide a theoretical basis for analyzing the cognitive processes implied in the teaching/learning of the sciences (Moreira, 1996). Subsequently, on the basis of this theory, Greca and Moreira (1998) endeavored to detect the type of mental representation that university students use when resolving problems and issues concerning the concept of the electromagnetic field.

Costa and Moreira (2001) insist on the construction of a proper mental model, beginning with the wording of a problem, as a necessary condition for solving it. Rodriguez-Palmero, Marrero-Acosta and Moreira (2001) show how essential it is to build mental models in order to understand how the living tissues of the brain function. Finally, several authors of this school sought in their works to delimit the theory of mental models, and to fit it within other theories of knowledge construction and the teaching/learning of science (Greca and Moreira, 2002a and 2002b; Moreira, Greca and Rodriguez-Palmero , 2002; Rodriguez-Palmero, 2004).

When focusing on problem solving, it is important to point out that other cognitive psychologists have also made use of mental models as cognitive structures that students create in the resolution processes. Thus, Anderson (1995) considers mental models to be the synthesis of declarative knowledge in a construct optimized for solving problems. We must remember that declarative knowledge is knowing "that"; i.e., it refers to the specific content or factual knowledge within a discipline or domain, and includes facts, concepts and principles. This author affirms that to solve problems it is necessary to restructure mental models and to cause them to operate, and for that, it is necessary to develop a solid base of declarative knowledge. Consequently, the development of mental models is crucial for success in problem solving.

Mayer (1992) proposes a cognitive model to explain problem solving. This cognitive model can be summarized in two main steps: translation and integration of the problem; and planning and implementation of the solution. In the first step, the problem-solver must transform the information in the statement, according to the knowledge available, into a mental model. The second step, which outlines a strategy for solving the problem, depends on the successful transformation of the problem into a proper mental model. During the planning of the resolution, the solver must assemble the information provided by the problem (including what the problem asks for), with what is stored in the working memory in the knowledge schematics. If you cannot make the assembly, you cannot get a strategy for resolution. In addition, this psychologist recommends teaching students to identify common resolution strategies for addressing various problems and contexts.

Those researching the teaching of science have also fixed their attention on the mental representations (mental models) that students construct when they try to solve a problem (Bodnar and Domin, 2000; Butel Gangoso, Brincones, and Gonzalez Martinez, 2001; Coleoni, Otero, Gangoso, and Hamity, 2001; Otero, and Elichiribehety Papini, 1998). Only the works of the last two groups mention the Johnson-Laird theory of mental models; however, all emphasize the relevance of forming proper mental models for correct problem solving. Bodnar and Domin (2000), indicate that students who are successful in solving chemistry problems, work out, on the average, more mental models that those who are not. In addition, the two groups of students differ from each other in the nature of their mental representations: those of the first are predominantly symbolic (contain symbols describing or approximating the physical reality), while those of the second are predominantly verbal (containing proposals, sentences or phrases).

These results are entirely consistent with those obtained by Greca and Moreira (1996 and 1998), who found that the best performance in the solution of problems in electromagnetism were students who had formed a mental model of the electromagnetic field similar to the conceptual model used by expert physicists. In contrast, students who worked only with isolated propositions (formulas, definitions and statements of laws), and who limited themselves to a mechanical application of these, had a lower f success rate.

A recent article (Portoles and Sanjose, 2007) presents other cognitive variables that have proved to be decisive in solving problems. Specifically, the work of Solaz-Portoles and Sanjose (2006) analyzes the role played by the variables *previous knowledge, research strategies* and *conceptual knowledge* (concepts and propositional structures in the long-term memory), in solving problems. The results obtained in three statistical analyses, correlations between variables; multiple regression analysis; and stepwise regression analysis, indicate that the three variables mentioned have a statistically significant influence on success in solving problems. In addition to these three variables, conceptual knowledge has proven to be the most important contributor in problem solving.

This article presents an experiment in which students with different previous knowledge solve problems after reading a text. The objective is to test the theory of mental models, and to analyze the role which previous knowledge plays in the construction and operation of these models.

In light of the theoretical grounds, the first hypothesis is that the more mental models needed at once for solving a problem, the harder it will be. The second hypothesis focuses on the subjects' previous knowledge: those with more previous knowledge will find it easier to solve problems, since they possess the knowledge structures (schematics) which allow them to develop and implement the mental models needed for the cognitive processes used in problem solving.

I. Methodology

1.1 Subjects

The research involved 85 first-year students(16 years old), from a public high school in the Camp de Turia district, in Valencia, Spain. Of these, 43 first entered high school during the 2001-2002 academic year, and the rest entered the following year (2002-2003). The entire group were taking Physics and Chemistry, and only two had not taken the previous elective course.

1.2 Materials

1.2.1 Test of previous knowledge

The aim of this test was to access the subjects' semantic memory structure or cognitive structure. This means that an attempt was made to measure the subjects' conceptual or propositional knowledge on the topic of *atomic models* at the beginning of the experiment. One of the most successful instruments used in the endeavor to reach the above objective, is the concept map (Novak and Gowin, 1999; Moreira and Buchweitz, 2000). However, from a practical point of view, the task of developing concept maps requires specific training. Moreover, analyzing and assessing these maps accurately, requires complicated and arduous work on the part of the evaluator, since indicators of integration and differentiation of concepts must be given, as well as statements of proposals (West and Pines, 1985).

Given the limitations imposed by the research (an excessive number of sessions could not be used because of involved students' loss of conventional classes; measures were needed to avoid a disproportionate amount of labor in correcting the high number of tests, etc.), it was decided that we should use a test that did not require previous training, and to leave the students a relatively wide margin of maneuverability in taking it.

In the test that was used (a much-simplified version of that proposed in the work of Hegarty-Hazel and Prosser [1991]), students were provided with a list of 15 concepts. These concepts had been previously selected by two Physics and Chemistry teachers, one of whom belongs the group of authors of this paper, after a detailed examination of the subject of *Atomic Models* (see Annex I). Using these concepts, subjects were asked to write from five to ten sentences of whatever length they wished, whether or not the concepts were on the list.

For evaluating the test, the teachers, working together, made a concept association map showing all the possible relationships between the 15 concepts ("internal concepts"). Also included in the evaluation of the test was a list of eight "external concepts" (see Annex I), because of their relevance to the subject matter.

The concept association map was used to give an account of the relationship between pairs of internal concepts (propositions), in the subjects' protocols. These relationships were counted if they were correct and if they conformed to any of those shown on the association map, regardless of how they were written. In addition, the internal concepts counted for more than the external ones, when the latter were part of correct propositions.

If we begin with the suggestion made by some researchers (Novak, 1988a, 1988b; Chi, Feltovich and Glaser, 1981), that the difference between experts and novices is that the former have more concepts incorporated into their cognitive structure, and that the extent and quality of their propositional links is greater, it is reasonable to admit that previous knowledge should be directly proportional to the number of concepts, as well as the number of relationships between those concepts. Therefore, a good quantifier for proof of previous knowledge (PK) might be the product of the total concepts (internal and external) and the relationships between them. However, there is a high correlation between these, since the number of relationships increases with the number of concepts.

If we assume that the dependency between the number of relationships and the number of concepts is linear (if it is of a higher order, reasoning would have the same value), then the product of these two measurements has a quadratic dependency with the number of concepts. This quadratic dependency can be linear if you take the square root of the product, rather than the product directly. In general, the square root improves the effect of the product by eliminating much of the cumulative effects, because of the correlation. Ultimately, an appropriate quantifier for this test proves to be the square root of the product of the concepts and relationships for each subject analyzed:

$$P.K. = \sqrt{concepts\ x\ relationships}$$

.

Once these measurements are obtained for all the subjects, the representative value of the group will be the arithmetic mean of these quantities, as well as its standard deviation. The protocols of previous knowledge were corrected separately by two Physics and Chemistry teachers (one of them belongs to the group of authors of this paper), and discrepancies were resolved by mutual agreement.

1.2.2 Problem Solving Test

The purpose of this test was to evaluate the ability of the subjects to transfer and apply their knowledge to new contexts or situations. Prepared for this test was an open questionnaire of six items on *Atomic Models* (see Annex II). Five of these can be regarded as conceptual (Items 2, 3, 4, 5 and 6) and one as algorithmic (Item 1).

Considered as algorithmic problems were those which involved only the solving of equations, application of rules, and the performing of calculations. Conceptual problems are those that require an understanding of concepts and inferential reasoning.

Quantification of the test was carried out by a previous categorization of the students' answers, which led to a single category of correct answers per item submitted, and a subsequent valuation of the presence/absence of the correct answer as 1/0.

Table I shows the conceptual content and the possible mental models to be implemented in the proper resolution of each of the items in this test of problem solving.

Table I. Conceptual content of each item, and mental models to be used, as a minimum, in each

Item	Conceptual Content	Mental Models
1	-In neutral atoms, the atomic number (subscript) is equal to the number of protons and electrons. -The mass number (superscript) is equal to the number of protons plus the number of neutrons.	-No mental model. Only propositional representation. Simply remember: a) the definitions of atomic number and mass, and b) how to show an isotope (atom).
2	-Rutherford's Experiment consists of launching positively-charged particles against a thin gold foil. -The gold foil in Rutherford's Experiment is made up of a network of gold atoms. -The protons are concentrated in the atomic nucleus. -Like-charged particles repel each other. -When a charged particle moves past another like-charged particle, its trajectory is changed: the closer it comes to the other particle, the greater the repulsion force and diversion of the trajectory.	-Model of material composed of atoms. -Rutherford's atomic model: protons concentrated in the nucleus, together with the neutrons and electrons which revolve around it. -Model of the interaction between charges, and its effect on trajectories. -Model of Rutherford's Experiment (launching of positively-charged particles against atoms.)
3	-Protons are concentrated in the atomic nucleus. -Atomic nucleus is very small, as compared to the atom itself. -The electrons move around the nucleus in a large, empty space. -The protons are not readily accessible. -The electrons are readily accessible.	-Rutherford's atomic model. -Model of how to remove or add electrons to the atom. -Model of accessibility and manipulation of the electrons and protons of the nucleus.
4	-Rutherford's Experiment consists of launching positively-charged particles	-Model de materials composed of atoms. -Rutherford's atomic model.

	against a thin gold foil. -The gold foil in Rutherford's Experiment is made up of a network of gold atoms. -The protons are concentrated in the atomic nucleus. -Like-charged particles repel each other. -When a charged particle moves past another like-charged particle, its trajectory is changed: the closer it comes to the other particle, the greater the repulsion force and diversion of the trajectory.	-Model of the interaction between charges, and its effect on trajectories. -Model of Rutherford's Experiment. .
5	-Rutherford's Experiment consists of launching positively-charged particles against a thin gold foil. -The gold foil in Rutherford's Experiment is made up of a network of gold atoms. -The protons are concentrated in the atomic nucleus. -Like-charged particles repel each other. -When a charged particle moves past another like-charged particle, its trajectory is changed: the closer it comes to the other particle, the greater the repulsion force and diversion of the trajectory. -The electric charge by rubbing is acquired because of a gain or loss of the most external of the atom's particles: the electrons.	-Model of material composed of atoms. -Rutherford's atomic model. -Model of the interaction between charges, and its effect on trajectories. -Model of Rutherford's Experiment -Model of charge acquisition by rubbing. -Model of accessibility and manipulation of electrons and protons.
6	-The number of protons or atomic number identifies atoms of the same chemical element. -The number of electrons in an atom is not always the same: it may have gained or lost electrons. -The number of neutrons can be different for atoms of the same chemical element. -Isotopes are atoms of the same chemical element with a different number of neutrons.	-Model of a chemical element. -Model of an isotope. -Rutherford's atomic model. -Model of accessibility and manipulation of electrons. -Model of charge acquisition (by gaining or losing electrons.)

1.3 Procedure

Two sessions were used: the first, 30 minutes long, and the second, 55 minutes. In the first session, students were told that they were going to participate in a research project on the teaching of science, and they took the test on previous knowledge. In the second session, the text was distributed among the participating subjects. Next, the students were given 22 minutes in which to read the text, alter which the text was taken away and the problem-solving test applied (approximately 20 minutes). Between reading the text and taking the test, there was a three-minute period in which the students did distracting tasks. The time was sufficient for all students to finish the test.

II. Results

Table II shows the arithmetic mean and standard deviation of the cognitive variable, previous knowledge.

Table II. Descriptive statistics of the variable previous knowledge

Name of variable	Instrument	Type of variable	Arithmetic mean	Standard Deviation	Maximum points possible
Previous Knowledge (PK)	Test of conceptual and propositional knowledge	Independent (sq. root of the product of concepts and relationships)	9.3	3.4	31.4

Based on these results for the test on previous knowledge, the students were classified in two groups: high previous knowledge and low previous knowledge. The first group, a total of 43 subjects, were those who obtained a grade of 9.3 or higher on the test. The second group, 42 students, obtained a lower score.

Figure 1 shows the percentage of subjects who answered each item correctly.

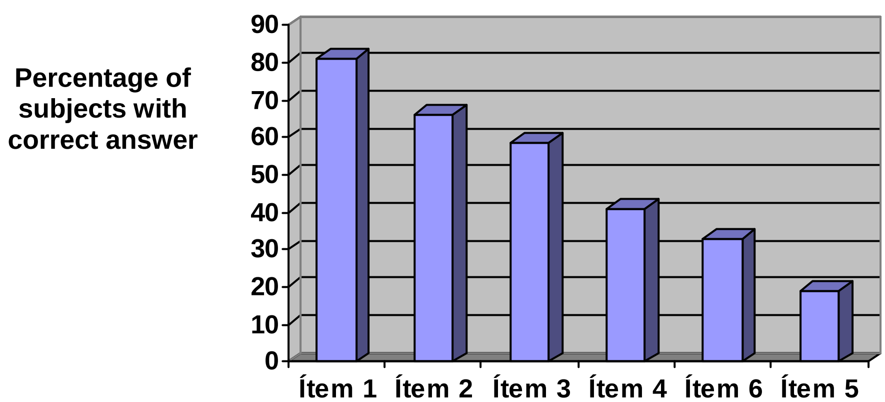

Figure 1. Percentage of subjects who answered each of the items correctly

Figure 2 shows the percentage of subjects who answered each item appropriately, based on their previous knowledge.

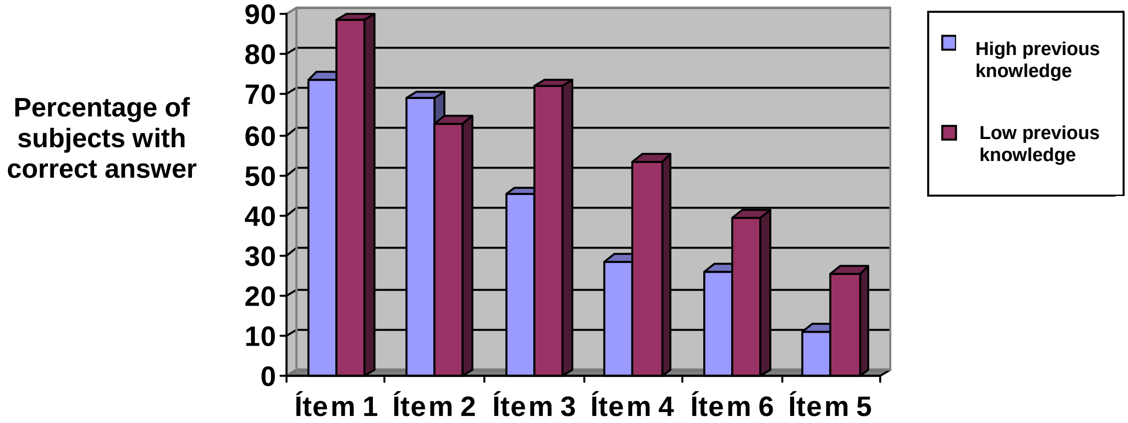

Figure 2. Percentage of subjects who answered each item correctly, according to their previous knowledge

Figure 3 illustrates the relationship *number of subjects with high previous knowledge/number of subjects with low previous knowledge*, for the subjects who answered each item correctly.

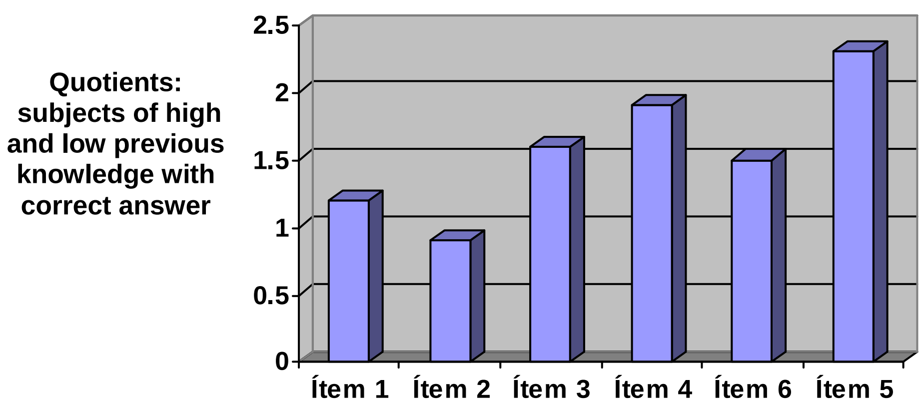

Figure 3. Quotients: number of subjects with high and low previous knowledge, who correctly answered each of the items

The application of the Chi-square test to groups of low and high previous knowledge on every item, based on 2x2 contingency tables constructed with subjects having high and low previous knowledge, and who answered the item correctly or incorrectly, generates significant differences ($p < 0.05$) only in Items 3 and 4. Specifically, it provides the following values: in Item 3, $\chi 2=6.33$, g.l.= 1, $p<0.05$; and in Item 4, $\chi 2=5.45$, g.l.=1, $p<0.05$. Consequently, only in these two items does the variable previous knowledge discriminate between individuals in connection with their success in solving such problems.

III. Discussion

Figure 1 demonstrates that the more mental models it is necessary to use, the more difficult is the solution of a problem, and the lower the percentage of subjects who solve it appropriately. Thus, Item 1 (algorithmic), which does not require a mental model for its resolution (see Table I), but only a propositional representation, registers a high success rate (81.1%). Items 2, 3 and 4 (percentage of success 65.9%, 58.8% and 41.2%, respectively) require for their resolution, the use of two to four mental models (see Table I). Finally, items 5 and 6 (with success rates of 18.8% and 33% respectively) in order to be solved, need to have in place at least six and five mental models, respectively (see Table I).

As may be seen, and in complete accord with the premises of the theory of mental models (Johnson-Laird y Bara, 1984) and with our first hypothesis, there is an inverse relationship between the minimum number of models involved in the proper resolution of a problem and the percentage of students who solve it correctly: the greater the number of mental models needed for the resolution of a problem, the lower the percentage of subjects who solve it correctly.

Obviously, the existence of any item whose results deviate from this tendency is attributable to factors not controlled in the experiment, or to students' need for some other mental model not considered in this analysis. Thus, Item 3 is more difficult than Item 2, which in principle requires fewer mental models. This case could be explained by the fact that the text students were given to read contained information which is an explicit response to Item 2, and which would allow subjects to reduce the number of mental models they needed to use.

Figure 2 and Figure 3 seem to suggest that as the problem increases in difficulty, the influence of previous knowledge in its resolution becomes more important. That is, the greater the previous knowledge, the greater the probability of successfully resolving difficult problems. However, the statistical test Chi square warns us that the variable previous knowledge generates significant differences among subjects with different previous knowledge only when the problems are neither very easy nor very difficult. This means that only in solving those problems which need just a few mental models (two to four) for their resolution, previous knowledge of the subjects makes a difference in solving these problems successfully. Therefore, this result qualifies the second hypothesis of the study and clearly defines the role of previous knowledge in solving problems.

Finally, those problems will be analyzed in which previous knowledge is not crucial for solving them correctly. Item 1, which is an algorithmic problem, requiring no mental model for its resolution, may be solved simply by the use of a mental propositional representation, which would justify the high number of students who successfully solved it, and the limited influence of previous knowledge. In contrast, items 5 and 6 (the most difficult and conceptual in nature), require to have in operation at least five to six mental models for their resolution. Given the limitations of processing capacity in the working memory, it may be that the demand for this working memory exceeds, in most cases, the said working memory's processing capacity (Johnstone and El-Banna, 1986; Johnstone, Hogg and Ziane, 1993; Niaz, 1987). This fact could explain why the majority of students participating in this study, regardless of prior knowledge, fail in solving Item 5 and Item 6 of the test. It may be that they are unable to process simultaneously the information needed to keep so many mental models running at the same time.

IV. Implications for teaching

Teachers should take into account their students' previous knowledge when designing instructional material. Once the deficiencies are known, teachers should act to provide the same level of knowledge to all their students before beginning the development of a teaching unit (a pre-learning sequence). This would allow students to address effectively the various learning activities, especially problem solving.

An essential recommendation for teachers: never skip making an assessment of the difficulty in the wording of a problem, based on to the number of mental models which must be activated for in solving it. This means that one must conduct a preliminary analysis of what students know: what information must be provided to students, to which students it is directed, at what stage of the curriculum is it to be used, etc. One particular type of problem should be noted: it typically occurs in the midst of a plethora of pedagogical texts, and is overly used in the evaluation of learning. It is the algorithmic problem. Algorithmic problems do not normally require any mental model on the part of the student, and consequently have little value as indicators of the understanding of concepts. However, this type of problem may be serviceable in a limited way as a tool in specific educational situations.

Finally, we must remember that the basis of adequate knowledge for problem solving, in addition to conceptual knowledge (also called declarative knowledge) consists of situational awareness, procedural knowledge and strategic knowledge (Ferguson-Hessler, De Jong, 1990). On the other hand, we can not ignore those strategies which have not been addressed here, but which are equally crucial in problem solving. These also must be the object of specific instruction: cognitive strategies for controlling our knowledge, and understanding or metacognitive strategies (Bell, Cuervo, Moya and Otero, 1998).

Translator: Lessie Evona York Weatherman

UABC Mexicali

References

Anderson, J. (1995). *Learning and memory: An integrated approach*. New York: Wiley.

Bodner M. G. & Domin, D. S. (2000). Mental models: The role of representations in problem solving in Chemistry. *University Chemistry Education, 4* (1), 24-30.

Buteler, L., Gangoso, Z., Brincones, I., & González Martínez, M. (2001). *Enseñanza de las Ciencias, 19* (2), 285-295.

Campanario, J. M., Cuerva, J., Moya, A., & Otero, J. (1998). La metacognición y el aprendizaje de las ciencias. In E. Banet & A. de Pro (Eds.), *Investigación e innovación en la enseñanza de las ciencias* (Vol I, pp. 36-44). Murcia: Editorial Diego Marín.

Chi, M.T.H., Feltovich, P J., & Glaser, R (1981). Categorization and representation of physics problem by experts and novices. *Cognitive Science, 5*, 121-151.

Coleoni, E. A., Otero, J. C., Gangoso, Z., & Hamity, V. (2001). La construcción de la representación en la resolución de un problema de física. *Investigaçoes em Ensino de Ciências, 6* (3). Retrieved September 20, 2006, from: http://www.if.ufrgs.br/public/ensino/revista.htm

Costa S. S. C. & Moreira, M. A. (2001). A resoluçao de problemas como um tipo especial de apredizagem significativa. *Caderno Catarinense de Ensino de Física, 18* (3), 263-277.

Ferguson-Hessler, M.G. & De Jong, T. (1990). Studying physics texts: Differences in study processes between good and poor performers. *Cognition and Instruction, 7*, 41-54.

García-Madruga, J. A., Gutiérrez, F, Carriedo, N., Moreno, S., & Johnson-Laird, P. N. (2002). Mental models in deductive reasoning. *The Spanish Journal of Psychology, 5* (2), 125-140.

Greca, I. M. & Moreira, M. A. (1996). Un estudio piloto sobre representaciones mentales, imágenes, proposiciones y modelos mentales respecto al concepto de campo electromagnético en alumnos de física general, estudiantes de posgrado y físicos profesionales. *Investigaçoes em Ensino de Ciências, 1* (1). Retrieved November 1, 2006, from: http://www.if.ufrgs.br/public/ensino/revista.htm

Greca, I. M. & Moreira, M. A. (1998). Modelos mentales y aprendizaje de física en electricidad y magnetismo. *Enseñanza de las Ciencias, 16* (2), 289-303.

Greca, I. M. & Moreira, M. A. (2002a). Mental, physical, and mathematical models in the teaching and learning of physics. *Science Education, 86* (1), 106-121.

Greca, I. M. & Moreira, M. A. (2002b). Além da detecçao de modelos mentais dos estudantes. Uma proposta representacional integradora. *Investigaçoes em Ensino de Ciências, 7* (1). Retrieved October 22, 2006, from: http://www.if.ufrgs.br/public/ensino/revista.htm

Hegarty-Harzel, E. & Prosser, M. (1991). Relationship between students' conceptual knowledge and study strategies, part 1: students learning in physics. *International Journal of Science Education, 13*, 303-312.

Johnson-Laird, P.N. (1983). *Mental models: Towards a cognitive science of language, inference, and consciousness*. Cambridge: Cambridge University Press.

Johnson-Laird, P. N. (1990). Mental models. In M. Posner (Ed.), *Foundations of cognitive science* (pp. 469-499). Cambridge, MA, United States: MIT Press.

Johnson-Laird, P. N.(1996). Images, Models and Propositional Representations. In M. De Vega, M. J. Intons-Peterson, P. N. Johnson-Laird, M. Denis, & M. Marschark (Eds.), *Models of Visuospatial Cognition* (pp 90-127). Oxford: Oxford University Press.

Johnson-Laird, P. N. (2000). The current state of the mental model theory. In J. A. García-Madruga, N. Carriedo, & M. J. González Labra (Eds.), *Mental models in reasoning* (pp.16-40). Madrid: Universidad Nacional de Educación a Distancia.

Johnson-Laird, P. N. & Bara, B. G. (1984). Syllogistic inference. *Cognition, 16*, 1-62.

Johnstone, A. H. & El-Banna, H. (1986). Capacities, demands and processes –a predictive model for science education. *Education in Chemistry, 23*, 80-84.

Johnstone, A. H., Hogg, W. R., & Ziane. M. (1993). A working memory model applied to physics problem solving. *International Journal of Science Education, 15*, 663-672.

Mayer, R. E. (1992). *Thinking, problem solving and cognition*. New York: Freeman.

Moreira, M. A. (1996). Modelos mentais. *Investigaçoes em Ensino de Ciencias, 1* (3). Retrieved September 30, 2006, form: http://www.if.ufrgs.br/public/ensino/revista.htm

Moreira, M. A. & Buchweitz, B. (2000). *Novas estratégias de Ensino e Aprendizagem*. Lisboa: Plátano.

Moreira, M. A., Greca, I. M., & Rodríguez Palmero, M. L. (2002). Modelos mentales y modelos conceptuales en la enseñanza y aprendizaje de las ciencias. *Revista Brasileira de Pesquisa em Educaçao em Ciências*, *2* (3), 37-57.

Níaz, M. (1987). Relation between M-Space of students and M-Demand of different items of General Chemistry and its interpretation based upon the neo-Piagetian theory of Pascual-Leone. *Journal of Chemical Education, 64*, 502-505.

Novak, J. D. (1988a). Constructivismo humano: Un consenso emergente. *Enseñanza de las Ciencias, 6*, 213-223.

Novak, J. D. (1988b). Learning science and the science of learning. *Studies in Science Education, 15*, 77-101.

Novak, J. D. & Gowin. D. B. (1999). *Aprendiendo a aprender*. Barcelona: Martínez Roca.

Otero, M. R., Papini, C., & Elichiribehety, I (1998). Las representaciones mentales y la resolución de un problema: Un estudio exploratorio. *Investigaçoes em Ensino de Ciencias, 3* (1). Retrieved October 5, 2006, from: http://www.if.ufrgs.br/public/ensino/revista.htm

Rodríguez-Palmero, M. L. (2004). La teoría del aprendizaje significativo. In A. J. Cañas, J. D. Novak, & F. M. González (Eds.), *Proceedings of the First Conference on Concept Mapping.* Pamplona, España.

Rodríguez-Palmero, M. L., Marrero Acosta, J., & Moreira, M. A. (2001). La teoría de modelos mentales de Johnson-Laird y sus principios de aplicación con modelos mentales de célula en estudiantes del curso de orientación universitaria. *Investigaçoes em Ensino de Ciencias, 6* (3). Retrieved October 20, 2006, from: http://www.if.ufrgs.br/public/ensino/revista.htm

Santamaria, C., García-Madruga, J. A., & Carretero, M. (1996). Beyond belief bias: Reasoning from conceptual structures by mental models manipulation. *Memory & Cognition, 24* (2), 250-261.

Solaz-Portolés, J. J. & Sanjosé, V. (2006). ¿Podemos predecir el rendimiento de nuestros alumnos en la resolución de problemas? *Revista de Educación, 339*, 693-710. Retrieved October 2, 2006, from: http://www.revistaeducacion.mec.es/re339.htm

Solaz-Portolés, J. J. & Sanjosé, V. (2007). Cognitive variables in science problem solving: A review of research. *Journal of Physics Teacher Education Online, 4* (2), 25-33. Retrieved May 5, 2007, from: http://www.phy.ilstu.edu./jpteo/issues/jpteo4(2)win07.pdf

West, L. H. T. & Pines, A. L. (1985). *Cognitive structure and conceptual change*. New York: Mc Millan

Annex I. Internal and external concepts for the Test on Previous Knowledge

Internal concepts (those which are provided to the student): Subject matter, atom, atomic model, experiment, Rutherford, mass, charge, nucleus, particle, electron, proton, atomic number, element, neutron, mass number.

External concepts: Hydrogen atom, periodic table, positive charge, negative charge, neutral charge, size, vacuum, isotope.

Annex II. Problem-solving test

Indicates what subatomic particles are present in the atom $^{27}_{13}Al$.

Why is it that some positive particles (the projectiles) show greater divergence than others in Rutherford's Experiment?

Why is it easier to remove electrons from an atom, or add electrons to it, than to remove or add protons?

If, in Rutherford's Experiment, negatively-charged atoms had been used as projectiles, and if the results had been the same, what model would you propose for the atom?

Using the model you have just proposed, how would you explain an experiment of electrification by rubbing?

If one atom has 6 protons, 6 electrons and 6 neutrons, and another atom has 6 protons, 5 electrons and 8 neutrons, are they both atoms of the same chemical element? Why?